Extreme Dependence in International Stock Markets*

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April 2009

Abstract

This paper investigates the structure and degree of extreme dependence in international equity markets using carefully selected tools from the theory of copulas. We examine both the static and dynamic dependence via unconditional and conditional copulas. We find significant asymmetric tail dependence in equity markets, with the overall larger lower tail dependence than upper tail dependence. Moreover, in Europe and East Asia but not in North America, the extreme dependence is time-varying in both its structure and degree. Our results also indicate a higher intra-continental than inter-continental tail dependence. Our findings have important implications in global risk management strategies.

*The author would like to thank Loran Chollete, John Knight, Leo Michelis, Stephen Sapp,Yuguo Sun, Tony Wirjanto, and participants of 2008 CEA meeting for valuable comments and suggestions. Financial support from Ryerson University is also acknowledged.

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JEL classification: C14, C51, G01, G15, G32

Keywords: Copulas; Tail dependence; Time varying dependence; International financial markets; Risk diversification.

1 Introduction

The interdependence between international stock markets strongly affects risk management across countries. The interdependence between stock markets bears on the joint distribution and diversification of international portfolios and thus their joint risk and overall returns. Therefore, it is crucial for global investors to understand and measure the structure and degree of dependence in the international markets.

This paper investigates the dependence between international stock markets using a relatively new but fast developing approach-copulas, which allows us to examine both the structure and the degree of dependence and how they evolve over time.

Three approaches have been mainly used to study dependence between financial markets: a joint distribution approach, a conditional correlation approach and a copula approach. A joint distribution approach is usually limited to a joint Gaussian distribution and the correlation coefficient is used to measure dependence. However, the linear correlation does not give information of the structure of the dependence and is only applicable to elliptical marginal distributions, which are not true for many asset returns. Thus it is a rather limited measure of dependence. More detailed study of the limitation and pitfalls of linear correlation appears in Embrechts et al. (2002).

The second approach is to use conditional correlations. This approach measures correlations by taking into consideration the conditional information, making it more complete than unconditional linear correlation. However, Forbes and Rigobon (2002) point out that conditional correlations can change dramatically under different conditions, making the results often misleading and hard to interpret.

The third approach, a copula model, is a direct and flexible measure of dependence. It measures not only dependence strength but also dependence structure, in particular, and it can capture the dependence when markets are at extremes, ie, extreme dependence. Moreover, it can be applied to any distributions and can capture nonlinear dependence.

The copula approach has recently become the most significant new tool to address comovement between markets in a flexible way. Longin and Solnik (2001) use a Gumbel copula to study the dependence across international equity markets. Hu (2006) employs a mixture copula to investigate the dependence of international stock markets. Both studies focus on static copula models and use monthly data. Goorbergh (2004) and Jondeau and Rockinger (2006) use time-varying copulas for the dependence in the four major international stock markets. Rodriguez (2006) applies mixture copula models to investigate financial contagion in the East Asian stock markets and four Latin American stock markets during crises. These papers allow for dependence dynamics and use daily data. More application of copulas in market dependence and comovements can be found in Marshal and Zeevi(2002), Forbes and Rigobon (2002), Chollete et al. (2006), and Patton (2006).

In the current paper we explore the static and dynamic dependence of international stock markets using both the unconditional and conditional copula models. We examine the structure of dependence directly via a mixture copula model. We then allow the weights in the mixture copula model to vary conditionally, following an ARMA-like process. We also investigate the dynamics in the degree of dependence using the time-varying symmetrized Joe-Clayton copula of Patton (2006).

Our paper is similar to the previous literature in that it also examines dependence and co-movements of international financial markets. It is different from and contributes to the literature in the following ways. First, it studies dynamics of extreme dependence in international financial markets, not only in the degree of dependence but also in the structure of the dependence. Second, it uses carefully selected tools for complete measure of dependence from the theory of copulas. Third, it examines both unconditional and conditional copula models. Moreover, it examines a broader range of stock markets from the North America, Europe and East Asia.

We find significant asymmetric tail dependence in most of the return pairs, with lower tail dependence being greater than upper tail dependence on average. Furthermore, we find evidence that the dependence structure changes with the market status: the weight on the lower tail dependence increases during market stresses or crashes but decreases with market booms. The degree of extreme dependence tends to become stronger in Europe and East Asia but not in North America (US-Canada pair). This finding provides evidence of increasing comovements in European and East Asian regions. The absence of time-varying of dependence in the US-Canada markets reflects the fact that these two markets have long remained co-moving together at a high and stable level. Our results also indicate that dependence is higher between market pairs from the same continent than from different continents, which might reflect a higher market integration within the continent than across different continents.

Our findings have a number of important implications. First, the asymmetric tail dependence confirms the previous finding that international stock markets are more likely to go into a downturn, or even a crisis, together than into a boom together. Thus diversification across international markets would have very limited use since these markets are likely to suffer loss at the same time. Second, the time varying in both the structure and degree of the tail dependence implies that investors need to adjust their portfolios in a timely fashion. Third, diversification is more useful across continents than within a continent.

The paper is organized as follows. Section 2 presents the methodology used and Section 3 describes data and discusses empirical results with Section 4 offering a conclusion.

2 The Methodology

2.1 The copula concepts and measures of dependence

A copula is a function that links the marginal distributions into a joint distribution. The marginal distributions in a copula are uniformly distributed on the interval [0,1]. The most important result in copula theory is known as Sklar's theorem. For simplicity, we consider the bivariate case.

Sklar's Theorem: Let $F_{XY}(.)$ be a joint distribution function with margins $F_X(.)$ and $F_Y(.)$. Then there exists a copula C(.) such that for all x_t, y_t in R,

$$F_{XY}(x, y; \theta_x, \theta_y, \theta_c) = C(F_X(x; \theta_x), F_Y(y; \theta_y), \theta_c). \tag{1}$$

If $F_X(.)$ and $F_Y(.)$ are continuous, then $C(.)$ is unique; otherwise, $C(.)$ is uniquely determined on Range$F_X \times$RangeF_Y. Conversely, if $C(.)$ is a copula and $F_X(.)$ and $F_Y(.)$ are the marginal cumulative distribution functions, then the function $F_{XY}(.)$ defined by (1) is a joint cumulative distribution function with margins $F_X(.)$ and $F_Y(.)$.

By Sklar's theorem, a joint distribution can be decomposed into its univariate marginal distributions and a copula, which captures the dependence structure between the variables X and Y. As a result, copulas allow us to model the marginal distributions and the dependence structure of multivariate random variables separately.

Different copulas usually represent different dependence structures with the so called association parameters θ_c indicating the strength of the dependence. Some commonly used copulas in economics and finance include the bivariate Gaussian copula, the student-T copula, the Gumbel copula, and the Clayton copula.

Copulas have several desirable properties. One key property is that they are invariant under increasing and continuous transformations. This property is useful, as such transformations are commonly used in economics and finance. For example, the copula is invariant to logarithmic transformation of variables. This is not a property of the correlation coefficient, which is invariant only under linear transformations. Another property of copulas is that they provide complete information about the structure of dependence among random variables over their whole range of joint variation and not just over certain portions of it. Third, unlike correlation, copulas do not require elliptically distributed random variables of the marginals. As a result, they are especially useful when modeling the dependence between asset returns (especially from high frequency data). Fourth, copulas allow us to separately model the marginal behavior and the dependence structure. This property gives us more options in model specification and estimation. Finally, a useful property of copulas, which we exploit in this study, is that they provide analytic measures of dependence in the tails of the joint distribution.

Based on the selected copulas, we can define two alternative nonparametric measures of dependence between the two variables, namely the Spearman's ρ and Kendall's τ rank correlation coefficients. Unlike the simple correlation coefficient, these rank correlations do not require a linear relationship between the variables. For this reason, they are commonly studied with copula models. The relationship between Spearman's ρ measure and copulas can be expressed as:

$$\rho = 12 \int_0^1 \int_0^1 uv dC(u,v) - 3 \tag{2}$$

Kendall's τ, for variables X and Y, is defined as the difference between the probability of the concordance and the probability of the discordance. Specifically, the higher the τ value, the stronger the dependence between X and Y. The relationship between Kendall's τ measure and copulas can be stated as:

$$\tau = 4 \int_0^1 \int_0^1 C(u,v) dC(u,v) - 1 \tag{3}$$

From the above expression, we see that the Kendall's τ measure does not depend on the marginal distributions.

A more informative dependence measure based on copulas is tail dependence, which is used to measure co-movements of variables in extreme situations. Tail dependence measures the probability that both variables are in their lower or upper joint tails. Intuitively, upper (lower) tail dependence refers to the relative amount of mass in the upper (lower) quantile of their joint distribution. Because tail dependence measures are derived from copula functions, they possess all the desirable properties of copulas mentioned above. The lower (left) and upper (right) tail dependence coefficients are defined as

$$\lambda_l = \lim_{u \longrightarrow 0} Pr[F_Y(y) \leq u | F_X(x) \leq u] = \lim_{u \longrightarrow 0} \frac{C(u,u)}{u}, \tag{4}$$

$$\lambda r = \lim_{u \longrightarrow 1} Pr[F_Y(y) \geq u | F_X(x) \geq u] = \lim_{u \longrightarrow 1} \frac{1 - 2u + C(u,u)}{1-u}, \tag{5}$$

where λ_l and $\lambda r \in [0,1]$. If λ_l or λr are positive, X and Y are said to be left (lower) or right (upper) tail dependent; see Joe (1997) and Nelson (1999).

2.2 The mixture copula model

By copula theory, a linear combination of copulas gives a new copula. To study the dependence structure between variables, we compose a new copula by combining copulas with different tail dependence properties. That is, a mixture of a Clayton and a survival Clayton copula:

$$C_M(u,v) = wC_C + (1-w)C_{SC}, \text{ where } u = F_X(x) \text{ and } v = F_Y(y), \tag{6}$$

where C_C, and C_{SC} are the Clayton and survival Clayton copula to capture the left and right tail dependence respectively, $0 \leq u, v \leq 1$ and $0 \leq w \leq 1$. The weight of each copula would reflect the dependence structure. A higher value (greater than 0.5) of w indicates a stronger left tail dependence than right tail dependence. The function form of each copula in the mixture copula is as follows:

$$\begin{aligned} C_C(u,v) &= (u^{-\theta_1}+v^{-\theta_1}-1)^{-1/\theta_1} \\ C_{SC}(u,v) &= u+v-1+[(1-u)^{-\theta_2}+(1-v)^{-\theta_2}-1]^{-1/\theta_2} \end{aligned}$$

where $0 \leq u, v \leq 1$ and θ_1, $\theta_2 > 0$.

2.2.1 The SJC copula model

To examine the degree of dependence, we adopt the symmetrised Joe Clayton (SJC) copula used in Patton (2006). The SJC copula is a modification of the so called "BB7" copula of Joe (1997). It is defined as

$$\begin{aligned} & C_{SJC}(u,v|\lambda r,\lambda_l) \\ = \;& 0.5\times(C_{JC}(u,v|\lambda r,\lambda_l)+C_{JC}(1-u,1-v|\lambda_l,\lambda r)+u+v-1), \end{aligned} \tag{7}$$

where $C_{JC}(u,v|\lambda r,\lambda_l)$ is the BB7 copula (also called Joe-Clayton copula) defined as

$$\begin{aligned} & C_{JC}(u,v|\lambda r,\lambda_l) \\ = \;& 1-(1-\left\{\left[1-(1-u)^k\right]^{-r}+\left[1-(1-v)^k\right]^{-r}-1\right\}^{-1/r})^{1/k}, \end{aligned} \tag{8}$$

$$\text{with } k = 1/log_2(2-\lambda r) \text{ and } \; r=-1/log_2(\lambda_l)$$

where λ_l and $\lambda r \in (0,1)$. By construction, the SJC copula is symmetric when λ_l=λr. This copula is very flexible since it allows for both asymmetric upper and lower tail dependence and symmetric dependence as a special case.

2.2.2 Dynamics of the dependence

To examine the possibility of dynamic or time-varying tail dependence in the data, similar to that in Patton (2006), we estimate the following ARMA-type process for the weight parameter w_t in the mixture copula model :

$$w_t = (1 + \exp(-h_t))^{-1}, \tag{9}$$

$$h_t = h_0 + bh_{t-1} + a \sum_{j=1}^{p} |u_{t-j} - v_{t-j}|; \tag{10}$$

and for the tail dependence parameters $\lambda_{l,t}$ and $\lambda_{r,t}$:

$$\lambda_{l,t} = (1 + \exp(-h_{l,t}))^{-1}, \quad \lambda_{r,t} = (1 + \exp(-h_{r,t}))^{-1}, \tag{11}$$

$$h_{l,t} = h_{l,0} + b_l h_{l,t-1} + a_l \sum_{j=1}^{p} |u_{t-j} - v_{t-j}|, \tag{12}$$

$$h_{r,t} = h_{r,0} + b_r h_{r,t-1} + a_r \sum_{j=1}^{p} |u_{t-j} - v_{t-j}|. \tag{13}$$

The dynamic models contain an autoregressive term designed to capture persistence in dependence and a variable which is a mean absolute difference between u and v. The latter variable is positive when the two probability integral transforms are on the opposite side of the extremes of the joint distribution and close to zero when they are on the same side of the extremes. The logistic transformation of the ARMA process guarantees that the weight and tail dependence parameters lie in the [0,1] interval.

The focus of this paper is on the dependence between the returns of indices from different markets. This dependence is fully modeled by the copula models. To avoid any distortion of the parametric assumption of the marginal distributions, we do not specify any parametric form for the marginal distributions of the index returns. We use the empirical cumulative distribution function (ECDF) for the margins of each return series. As a result, our approach is a semiparametric approach, with a nonparametric form for the margins and a parametric form for the joint distribution.

After computing the empirical cumulative distribution functions of the return series (u and v in the copula function), we use a maximum likelihood approach to estimate the copula models. Under standard regularity conditions the ML estimator is consistent, asymptotically efficient, and asymptotically normal. The log likelihood function is the log of the copula density function.

3 The data and empirical results

3.1 The Data

We investigate the interdependence between eight stock indices from north America, Europe and East Asia. The labels are US for the S&P 500, CA for the TSX composite index, UK for the Financial Times 100 stock index, GM for the Deutsche Aktien Index, FR for the French Cotation Automatique Continue index, JP for the NIKKEI 225 stock average index, KO for the Korea SE composite index, and HK for the Datastream-computed Hong Kong stock market index. All indices are in US dollars.

Our sample covers the period from August 3, 1990 to January 31, 2008. All data are from Datastream, sampled at a daily frequency. The index returns are computed as the log difference between the stock index at time t and t-1, multiplied by100. Data during holidays are deleted to avoid holiday impact.

Table 1 summarizes the descriptive statistics for the return series. The standard deviation is much higher than the absolute value of the mean, reflecting highly volatile stock markets. The excess kurtoses for all return series are greater than zero, ranging from 2.5 to 9.3 and their skewness is not zero. Thus the stock returns reflect typical features of fat tail and skewed distribution, showing evidence of not-normal returns, and linear correlation is not suitable for measuring their dependence since it is only defined on elliptical distributions.

Table 2, Table 3 and Table 4 present the three conventional measures of dependence: linear correlation, Spearman rank correlation and Kendall's tau rank correlation. Since the East Asian markets are about 12 hours ahead of the north America markets, we also include the lagged US and CA returns. The linear correlations between the European pairs are strongest, ranging from 0.68 to 0.77, followed by the US-CA pair, the pairs between the US market and European markets, and the East Asian pairs. The weakest correlations are between the north American and East Asian pairs. The correlations between the contemporary US market and East Asian markets are smaller than the correlations between the one-day lag of the US returns and East Asian returns, providing evidence that the US market influences the East Asian markets more than the other way around. On the other hand, the correlation between the one-day lagged Canadian market and the East Asian markets is smaller than that of the same day. Thus the Canadian market does not seem to lead the East Asian market. This could be due to the fact that Canada has a small open economy.

Consequently, we use one-day lagged US returns but same-day Canadian returns when examining the dependence between the North American and East Asian markets. The results from Spearman rank correlation and Kendall's tau correlation exhibit a pattern similar to that of the linear correlation. However, they are smaller than the linear correlations for the same pair in general.

3.2 Results for the dependence structure

For the mixture copula model defined in (6), we find the following interesting results presented in Table 5.

First, the copula association parameters θ_1, θ_2 and the weight parameter w are strongly statistically significant (most at 1% level, a few at 5% level) except for the FR-JP pair and the CA-JP pair (significant at 10% level). This result indicates that international stock markets are dependent at both the left and the right tails of the joint distribution. In other words, international markets tend to move to the extremes (stress or boom) together.

Second, the weight on the Clayton copula, w, is greater than 0.5 for all pairs except the US-KO, FR-KO, and CA-HK pairs. We then formally test the null hypothesis of $w \geqslant 0.5$ against the alternative hypothesis of $w < 0.5$ using a t test. From the p values of the test, we can not reject the null hypothesis for any pairs except the US-KO, FR-KO, and CA-HK pairs. The higher weight on the Clayton copula indicates that the dependence is biased to the left tail, making the dependence asymmetric. Therefore, it is more likely that international markets crash together than boom together.

Third, judging from the log likelihood and the AIC criteria, we find that the European pairs have the strongest dependence among all pairs. This result is consistent with other measures of dependence presented in Section 2 and reflects a higher degree of market integration in the European region. The North American markets are more dependent with the European markets than with the East Asian markets, which is shown in the higher parameter values and a higher log likelihood and a lower AIC criteria. Meanwhile, the European markets are more dependent with the North American markets than with the East Asian markets. On the other hand, the East Asian markets are more dependent with the European markets than with the US markets.

Finally, markets are more dependent within the continent than across continents. For example, the Japanese market is much more dependent with the Hong Kong and Korean markets than with the US, UK, German and French markets, while the German market is much more dependent with the UK and French markets than with the US or East Asian markets. This might be related to a stronger trading, economic and financial relationship and more similarities in the market structure within the continent than across continents. It also implies a higher degree of market integration within a continent than across continents.

Next, we investigate the dynamics of the structure of dependence. We examine these dynamics by studying how the weight parameter w changes overtime[1]. We focus on the pairs within the same continents, which have been found to have relatively stronger tail dependence in the static model. The result is presented in Table 6. The copula parameters

[1] The lags of p are set to 10 in all pairs except for the JP-HK pair, where p is chosen to be 5.

are still statistically significant, confirming the tail dependence found in the static model. The parameters that control the variation of the weight parameter are significant for most of the pairs (except for the US-CA and UK-FR pairs), indicating the time varying of the weight parameter. Thus the dependence structure changes over time for most markets.

To visually represent how the dependence changes over time, we plot the dynamics of the weight parameter w in Figure 1. We notice that the average weight line (the dashed line) lies above the $w = 0.5$ line (the solid line), confirming on average the higher left than right tail dependence found in the static model. For the US-CA pair, w only fluctuates around the mean, showing no systematic change of the dependence structure. For the European pairs, w stays above 0.5 until the first quarter of 2003, when it decreases below 0.5 and stays low until the third quarter of 2007. Thus the dependence structure is biased more to the left tail up to 2003 and then biased to the right tail until late 2007. This pattern reflects the generally booming markets in Europe during 2003-2007. The phenomenon is strong and apparent in the GM-FR and GM-UK pairs, but much more weakly shown in the UK-FR pair. For the East Asian pairs, a sharp increase occurs in w from 1997 to 1998, showing that the dependence structure moves rapidly towards the lower tail dependence, reflecting the East Asian financial market crisis during that period. On the other hand, for the period of late 2003 to late 2007, as in the European markets, w decreases and stays low, indicating the dependence moves towards upper tail dependence, reflecting the booming of the East Asian markets during this period. Thus the dependence structure changes with the status of the markets in both the European and East Asian markets.

If we compare the dynamic mixture model with the static mixture model, we find that the AIC is reduced for each pair. Some reductions in AIC are quite large. For example, for the JP-KO pair, the AIC decreases from -354.0 for the static model to -469.4 for the dynamic model. As a result, the dynamic mixture model performs better than the static mixture model. Therefore, it is important to capture the dynamics of the dependence structure.

3.3 Results for the degree of dependence

The SJC copula model is used to directly examine the degree of tail dependence. We present the result for the static SJC model in Table 7. First the lower and upper tail dependence parameters are strongly significant for almost all pairs, the exception being the right tail dependence of the CA-KO pair. Second, the lower tail dependence parameter is significantly larger than the upper tail dependence in all pairs except for the GM-KO pair, indicating asymmetric tail dependence, with stronger dependence in the left tail than in the right tail. This finding is consistent with our findings in the mixture copula model and it implies that international diversification is of limited value since the pairs tend to be in the downturn together when diversification is most needed. Third, the pairs from Europe have the highest degree of tail dependence, again reflecting a higher degree of market integration than other regions. Finally, the degree of tail dependence is higher within a continent than across continents. For example, the lower tail dependence is 0.6179 for the GM-FR pair while it is 0.2125 for the US-GM pair. Therefore, the degree of market integration is higher within the continent than across continents.

Table 8 gives the result for the dynamic SJC model. The parameters for dynamics for the US-CA pair are not significantly different from zero, indicating insignificant change of the degree of the tail dependence in North America. For pairs in Europe and East Asia, most dynamic parameters are significant. Consequently the degree of tail dependence in these markets changes over time.

To illustrate how the degree of tail dependence evolves over time, we plot the lower and upper tail dependence coefficients in Figure 2. In each graph, the solid curve, dotted curve, and dashed line are for the lower tail dependence, upper tail dependence, and the mean of the difference between the lower and upper tail dependence respectively. For all pairs, we find that the mean line lies above the zero line. This result means that the average lower tail dependence is greater than the upper tail dependence, showing asymmetry of the tail dependence overall. For the US-CA pair, the tail dependencies fluctuate around the mean, showing no systematic dynamics. For the European pairs, the degrees of the tail dependence are higher and less volatile after 2003. This seems to show the evidence that the European markets move towards a higher degree of market integration after 2003. For the East Asian pairs, the degrees of the dependence tend to increase gradually. This might be the evidence that they are in the process of becoming more integrated.

Comparing the results of the static and dynamic SJC models, we find a reduction of the AIC in the dynamic SJC model, which performs better than the static SJC model. For the East Asian pairs, the reduction of AIC is almost doubled; for instance, for the HK-KO pair, the AIC decreases from -491.0 to -744.6.

Furthermore, the two copula models produce a similar tail dependence structure, log likelihood, and AIC. We interpret this result as the evidence of well performance of our models.

4 Conclusion

In this paper, we examine the extreme dependence of international stock markets via copula functions. We investigate the dependence using both the static and dynamic copula models. To investigate the structure of dependence, we compose a mixture copula model. Further, we allow the weight to change over time to capture the dynamics of the dependence structure. To analyze the degree of dependence, we use the Symmetrised Joe-Clayton copula of Patton (2006) and again allow the dependence degree to be time-varying. We use daily returns on the stock indices from North America, Europe and East Asia. We find significant asymmetric tail dependence in most of the return pairs, with lower tail dependence being greater than upper tail dependence. Furthermore, the dependence is time varying in both its structure and degree in the European and East Asian markets but not in the North American markets. The dependence dynamics reflect that the European and the East Asian markets are moving towards more integration. Finally, we also find that dependence is higher between market pairs from the same continent than from different continents. Our findings have important implications to risk management and diversification in the international stock markets.

References

[1] Chollete, L., de la Pena, V., Lu, C., 2006. The scope of internaitonal diversification: implications of alternative measures, working paper, NHH.

[2] Embrechts, P., Mcneil A., Straumann D., 2002. Correlation and dependence in risk management, *Properties and Pitfalls in Risk Management: Value at Risk and Beyond*, ed. M.A.H. Dempster, Cambridge University Press, Cambridge, pp. 176-223.

[3] Forbes, K., Rigobon, R., 2002. No contagion, only interdependence: measuring stock market co-movements, *Journal of Finance*, 57(5): 2223-2261.

[4] Goorbergh, R., 2004. A copula-based autoregressive conditional dependence model of international stock markets, working paper, De nederlandsche Bank.

[5] Hu, L., 2006. Dependence patterns across financial markets: a mixed copula approach, *Applied Financial Economics*, 16, 717-729.

[6] Joe, H., 1997. Multivariate models and dependence concepts, *Monographs on statistics and applied probability*, 73. London: Chapman and Hall.

[7] Jondeau, E., Rockinger, M., 2006., The Copula-GARCH model of conditional dependencies: an international stock market application, *Journal of International Money and Finance* 25 827–853.

[8] Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets, *Journal of Finance*, 56, 649–76.

[9] Marshal, R., Zeevi, A., 2002. Beyond correlation: extreme co-movements between financial assets, *Working Paper*, Columbia Business School.

[10] Nelson, R.B., 1999. *An introduction to copulas*, Springer, New York.

[11] Patton, A.L., 2006. Modelling asymmetric exchange rate dependence, *International Economic Review*, 47(2), 527-556.

[12] Rodriguez, J. C., 2006. Measuring financial contagion: a copula approach, *Journal of Empirical Finance* 14, 401-423.

[13] Sklar, A., 1959. Fonctions de répartition á n dimensions et leurs marges, *Publications de l'Institut de Statistique de l'Université de Paris*, 8, 229-231.

Table 1: Descriptive Statistics of Returns

Variable	N	Mean	Maximum	Minimum	Std Dev	Skewness	Kurtosis
r_us	2919	0.0210	5.5732	-7.1127	0.9986	-0.1607	3.8443
r_ca	2919	0.0200	4.7581	-6.9262	1.0020	-0.5398	4.1680
r_gm	2919	0.0193	9.331	-13.0580	1.4452	-0.1743	5.1935
r_uk	2919	0.0152	5.7656	-5.6577	1.0855	-0.0028	2.5004
r_fr	2919	0.0195	9.0585	-10.2874	1.333	-0.0790	3.6303
r_jp	2919	-0.0230	12.5711	-6.8306	1.6312	0.1522	2.9060
r_ko	2919	-0.0006	27.4437	-19.1055	2.8204	0.1361	7.7957
r_hk	2919	0.0560	15.5566	-13.5741	1.6178	-0.0437	9.2858

Table 2: Linear Correlations

	r_us	r_ca	r_uk	r_gm	r_fr	r_jp	r_ko	r_hk
r_us	1.0000	0.6123	0.3813	0.4068	0.3945	0.0891	0.0964	0.1235
r_ca	0.6123	1.0000	0.4376	0.4220	0.4332	0.1753	0.1583	0.2312
r_uk	0.3813	0.4376	1.0000	0.6590	0.7453	0.2632	0.2328	0.3066
r_gm	0.4068	0.4220	0.6590	1.0000	0.7748	0.2645	0.1934	0.3210
r_fr	0.3945	0.4332	0.7453	0.7748	1.0000	0.2508	0.2086	0.2953
r_jp	0.0891	0.1753	0.2632	0.2645	0.2508	1.0000	0.2730	0.3838
r_ko	0.0964	0.1583	0.2328	0.1934	0.2086	0.2730	1.0000	0.3431
r_hk	0.1235	0.2312	0.3066	0.3210	0.2953	0.3838	0.3431	1.0000
lr_us	-0.0404	0.1259	0.1956	0.1769	0.1958	0.2079	0.1730	0.2991
lr_ca	-0.0344	0.0829	0.1120	0.0944	0.1043	0.1658	0.1487	0.2559

Table 3: Spearman's Rho

	r_us	r_ca	r_uk	r_gm	r_fr	r_jp	r_ko	r_hk
r_us	1.0000	0.5400	0.3349	0.3212	0.3411	0.0771	0.0839	0.1236
r_ca	0.5400	1.0000	0.4138	0.3916	0.4161	0.1805	0.1512	0.2429
r_uk	0.3349	0.4138	1.0000	0.6307	0.7078	0.2473	0.2325	0.3001
r_gm	0.3212	0.3916	0.6307	1.0000	0.7159	0.2704	0.2023	0.3096
r_fr	0.3411	0.4161	0.7078	0.7159	1.0000	0.2290	0.2015	0.2875
r_jp	0.0771	0.1805	0.2473	0.2704	0.2290	1.0000	0.3000	0.3564
r_ko	0.0839	0.1512	0.2325	0.2023	0.2015	0.3000	1.0000	0.3700
r_hk	0.1236	0.2429	0.3001	0.3096	0.2875	0.3564	0.3700	1.0000
lr_us	-0.0586	0.1139	0.1382	0.1527	0.1504	0.1947	0.1531	0.2384
lr_ca	-0.0505	0.0629	0.0776	0.0771	0.0765	0.1597	0.1435	0.2131

Table 4: Kendall's Tau

	r_us	r_ca	r_uk	r_gm	r_fr	r_jp	r_ko	r_hk
r_us	1.0000	0.3845	0.2321	0.2223	0.2370	0.0519	0.0584	0.0836
r_ca	0.3845	1.0000	0.2865	0.2704	0.2884	0.1214	0.1041	0.1661
r_uk	0.2321	0.2865	1.0000	0.4610	0.5265	0.1676	0.1604	0.2069
r_gm	0.2223	0.2704	0.4610	1.0000	0.5425	0.1854	0.1409	0.2138
r_fr	0.2370	0.2884	0.5265	0.5425	1.0000	0.1560	0.1401	0.1974
r_jp	0.0519	0.1214	0.1676	0.1854	0.1560	1.0000	0.2109	0.2457
r_ko	0.0584	0.1041	0.1604	0.1409	0.1401	0.2109	1.0000	0.2620
r_hk	0.0836	0.1661	0.2069	0.2138	0.1974	0.2457	0.2620	1.0000
lr_us	-0.0402	0.0771	0.0941	0.1048	0.1029	0.1322	0.1055	0.1634
lr_ca	-0.0345	0.0427	0.0526	0.0524	0.0518	0.1076	0.0986	0.1457

Table 5: Static Dependence Structure—Mixture Copula

Pairs	θ_1	θ_2	w	lnL	AIC	P value for test H_0: w >=0.5 H_1: w <0.5
North America						
US-CA	1.1052(15.0495)	1.3236(11.2534)	0.5827(15.8239)	634.5212	-1263.0	1.0000
Europe						
UK-GM	1.3399(15.6463)	2.0771(11.1643)	0.5783(18.7593)	868.9	-1731.8	1.0000
UK-FR	1.8403(21.4293)	2.3578(14.2413)	0.6074(21.8330)	1167.7	-2329.4	1.0000
GM-FR	1.8437(17.9833)	3.0184(11.7035)	0.5827(21.6890)	1287.9	-2569.8	1.0000
East Asia						
JP-KO	0.3050(8.0569)	1.5001(5.6411)	0.7415(17.7722)	180.0	-354.0	1.0000
JP-HK	0.5575 (7.5146)	0.7291(4.9073)	0.6190(9.9244)	241.5	-477.0	1.0000
HK-KO	0.4358 (9.0764)	1.5441(7.1286)	0.6745(16.8638)	287.4	-568.8	1.0000
North America. - Europe						
US-GM	0.5446(5.8624)	0.6980(5.0294)	0.5609(7.5895)	250.8	-495.6	1.0000
US-UK	0.5238(5.6191)	0.8172(3.4671)	0.6291(7.5325)	243.5	-481.0	1.0000
US-FR	0.4507(8.0366)	1.0792(5.0292)	0.6458(11.9791)	257.1	-508.2	1.0000
CA-UK	0.6700(10.3720)	0.9834(6.6814)	0.6347(13.6260)	328.1	-650.2	1.0000
CA-GM	0.6507(9.2700)	0.7956(8.5799)	0.5384(10.9567)	295.2	-584.4	1.0000
CA-FR	0.7335(11.5357)	0.8401(11.9995)	0.5640(14.7141)	322.4	-638.8	1.0000
North America.–East Asia						
US-JP	0.2646(3.1643)	0.3838 (1.9666)	0.6282(3.994)	72.6	-139.2	0.2302
US-KO	0.7871(3.4420)	0.1346(4.0056)	0.2605(3.7901)	64.7	-123.4	0.0000
US-HK	0.3016(8.0400)	0.8391(3.9946)	0.7127(10.8870)	138.7	-271.4	1.0000
CA-JP	0.1849(5.9228)	*0.8233(1.8447)*	0.8539(12.4023)	53.8	-101.6	1.0000
CA-KO	0.1585(4.5423)	0.5859(2.2271)	0.7850(8.9092)	47.8	-89.6	0.8351
CA-HK	0.7186(3.7440)	0.2235(4.5724)	0.3739(4.1414)	107.1	-208.2	0.0011
Europ-East Asia						
GM-JP	0.2885(7.1943)	1.0401(3.2216)	0.7599(12.8507)	125.0	-244.0	1.0000
GM-KO	0.1504(4.4016)	1.4833(4.9764)	0.7876(19.2563)	88.3	-170.6	1.0000
GM-HK	0.6466(2.7770)	0.3730(2.0131)	0.5562(3.9912)	175.6	-345.2	0.1810
UK-JP	0.2910 (7.0807)	0.7007(3.5251)	0.7197(9.9567)	115.5	-225.0	1.0000
UK-KO	0.2339(6.1526)	0.9567(2.7156)	0.7594(10.5041)	100.0	-194.0	1.0000
UK-HK	0.3777(8.7968)	1.1612(3.0289)	0.7813(13.9067)	174.0	-342.0	1.0000
FR-JP	*0.4077(1.8505)*	*0.3234(1.7754)*	0.5532(2.4333)	101.3	-196.6	0.2186
FR-KO	1.0679 (2.9713)	0.1583(4.4171)	0.2479(3.7701)	82.9	-159.8	0.0000
FR-HK	0.3832 (7.1361)	0.9099(2.8118)	0.7880(13.4842)	154.9	-303.8	1.0000

Note: Numbers in brackets are t statistics.

Table 6: Dynamic Dependence Structure—Mixture Copula

Pairs	θ_1	θ_2	h_0	a	b	lnL	AIC
North America							
US-CA	1.0786**	1.3721**	-0.5256	4.1255	0.0735	636.1	-1262.2
	(15.6182)	(12.7104)	(-1.0054)	(1.4121)	(0.1646)		
Europe							
UK-GM	1.0752**	2.8490**	-0.0382**	0.2281*	0.9916**	891.8	-1773.6
	(18.1107)	(12.6414)	(-2.3635)	(2.1763)	(203.5983)		
UK-FR	1.6932**	2.7040**	-0.4931	6.0009	0.1154	1171.1	-2332.2
	(16.9540)	(11.2254)	(-1.1033)	(1.4654)	(0.2232)		
GM-FR	1.3741**	5.7579**	-0.1441**	1.1714**	0.9726**	1415.2	-2820.5
	(24.4371)	(18.0087)	(-2.9129)	(2.8355)	(105.3813)		
East Asia							
JP-KO	0.2079**	1.5301**	-0.2291**	0.9772**	0.9695**	239.7	-469.4
	(6.3465)	(10.0578)	(-2.7222)	(2.6774)	(85.8623)		
JP-HK	0.3768**	1.7246**	-0.1531**	0.7221**	0.9797**	271.8	-533.6
	(10.7550)	(8.6914)	(-2.5180)	(2.4721)	(136.6172)		
HK-KO	0.3024**	1.9344**	-0.1010**	0.4536**	0.9850**	347.1	-684.2
	(7.9247)	(9.9035)	(-2.5460)	(2.4499)	(160.9558)		

Note:. Numbers in brackets are t ratios. ** and * indicate significance at 1% and 5% level respectively.

Table 7: Static Tail Dependence—SJC Copula

Pairs	λ_l	λ_r	lnL	AIC	P value for Test $H_0 : \lambda_l >= \lambda_r$ $H_1 : \lambda_l < \lambda_r$
North America					
US-CA	0.4252 (24.1504)	0.3674(17.9063)	662.6	-1321.2	1.0000
Europe					
UK-GM	0.4969 (31.4554)	0.4544 (27.1508)	878.3	-1752.6	1.0000
UK-FR	0.5992 (50.4302)	0.5103 (28.5302)	1202.4	-2400.8	1.0000
GM-FR	0.6179 (53.8966)	0.5658 (44.2598)	1332.9	-2661.8	1.0000
East Asia					
JP-KO	0.1798 (7.3395)	0.1328 (5.2391)	174.1	-354.2	1.0000
JP-HK	0.2276 (9.8751)	0.1680 (6.7049)	247.5	-491.0	1.0000
HK-KO	0.2548 (11.5549)	0.2089 (8.4055)	283.7	-563.4	1.0000
North America-Europe					
US-UK	0.2219 (9.5331)	0.1802 (7.3734)	250.2	-496.4	1.0000
US-GM	0.2125 (9.2463)	0.2019 (8.7433)	266.1	-528.2	1.0000
US-FR	0.2136 (9.1476)	0.2046 (8.5340)	262.7	-521.4	1.0000
CA-UK	0.2832 (12.8832)	0.2115 (8.4748)	327.5	-651.2	1.0000
CA-GM	0.2326 (10.3341)	0.2291 (9.8833)	297.6	-591.2	1.0000
CA-FR	0.2642 (11.5282)	0.2267 (9.1691)	320.1	-636.2	1.0000
North America-East Asia					
US-JP	0.0764 (3.4114)	0.0514 (2.4527)	73.4	-142.8	1.0000
US-KO	0.0661 (3.0659)	0.0461 (2.3873)	64.7	-125.4	1.0000
US-HK	0.1362 (5.8073)	0.1085 (4.7270)	141.3	-278.6	1.0000
CA-JP	0.0623 (2.8605)	*0.0193 (1.2382)*	51.2	-98.4	1.0000
CA-KO	0.1000 (2601.1)	0.1000 (1496.8)	43.1	-82.2	0.0000
CA-HK	0.1214 (5.0529)	0.0719 (3.0669)	106.9	-209.8	1.0000
Europe-East Asia					
UK-JP	0.1127 (4.7183)	0.0857 (3.7355)	114.5	-225.0	1.0000
UK-KO	0.1029 (4.2755)	0.0826 (3.4502)	101.4	-198.8	1.0000
UK-HK	0.1982 (8.7398)	0.1004 (4.1018)	175.3	-336.6	1.0000
GM-JP	0.1358 (5.5374)	0.0835 (3.5173)	121.3	-238.6	1.0000
GM-KO	0.0735 (3.1536)	0.0785 (3.2827)	81.7	-159.4	0.0000
GM-HK	0.2049 (8.7617)	0.0949 (3.9806)	176.8	-349.6	1.0000
FR-JP	0.1151 (4.7466)	0.0644 (2.9448)	102.7	-201.4	1.0000
FR-KO	0.0859 (3.7346)	0.0636(2.8417)	82.5	-161.0	1.0000
FR-HK	0.1978 (8.5909)	0.0645 (2.7935)	154.6	-305.2	1.0000

Numbers in brackets are t ratios.

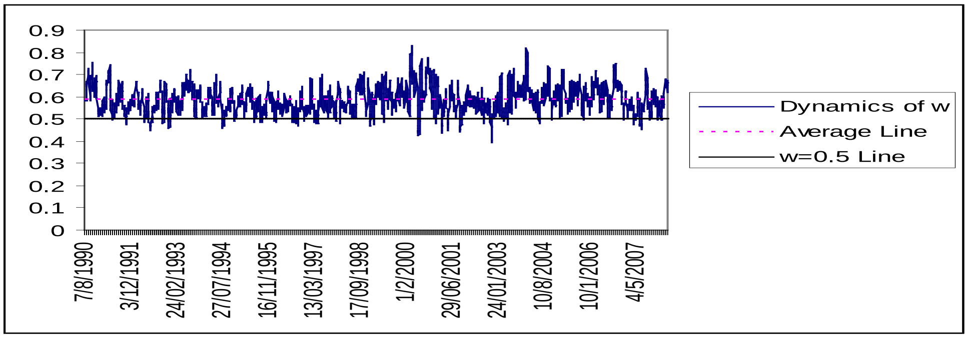
Table 8: Dynamic Tail Dependence --SJC Copula

Pairs	$h_{0,l}$	a_l	b_l	$h_{0,r}$	a_r	b_r	LnL	AIC
North America								
US-CA	-1.0654	0.2603	1.6550	0.2282	-1.7990	-1.1019	663.9	-1315.8
	(-1.1200)	(0.4532)	(0.7197)	(0.2196)	(-1.3916)	(0.4576)		
Europe								
GM-UK	1.9825**	-6.1059 **	-1.8071	-10.518**	-2.0911**	2.6791**	919.1	-1826.1
	(2.4364)	(-3.5775)	(-1.4925)	(4.1754)	(-4.8595)	(-2.5722)		
GM-FR	1.7253**	-10.3554**	0.4597**	1.2036**	-8.7370**	0.6928**	1510.1	-3008.2
	(6.4444)	(-7.3890)	(2.0575)	(3.6645)	(-5.2399)	(4.6444)		
UK-FR	1.2234**	-7.0151**	0.4526**	1.0014	-4.7034**	-0.3056	1254.9	-2497.8
	(10. 6997)	(-15.8109)	(6.2725)	(1.6973)	(-4.8066)	(-1.3806)		
East Asia								
JP-KO	-1.6086**	-2.2881**	4.0194**	1.4349	-12.9984**	-0.2950	260.5	-509.0
	(-14.1074)	(-5.2084)	(37.8461)	(1.1214)	(-3.0508)	(-0.1493)		
JP-HK	-1.8788**	-1.0471**	4.1532**	0.7570	-9.7303**	-2.8745	286.5	-561.0
	(-60.2677)	(-7.0318)	(100.0420)	(0.8216)	(-2.9015)	(-1.0549)		
HK-KO	-1.8224**	-1.4206**	4.1283**	-1.9229**	-1.1988*	4.2675**	378.3	-744.6
	(-30.7884)	(-4.8588)	(63.2069)	(-28.5593)	(-3.9916)	(87.1642)		

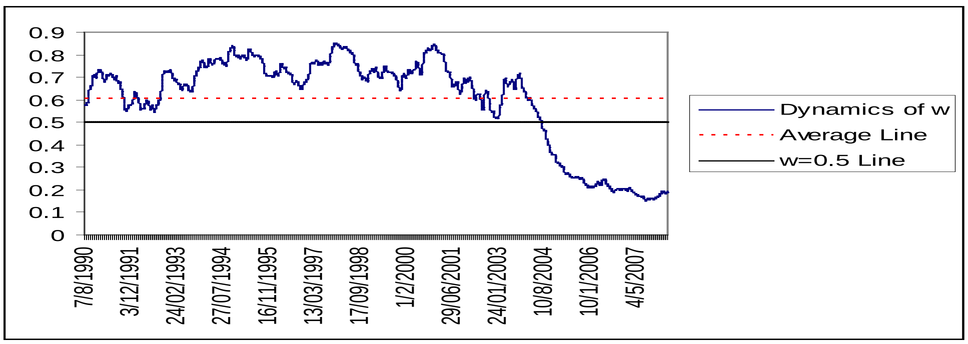
Note:. Numbers in brackets are t ratios. ** and * indicate significance at 1% and 5% level respectively.

Figure 1: Dynamics of the Weight for the Clayton Copula w

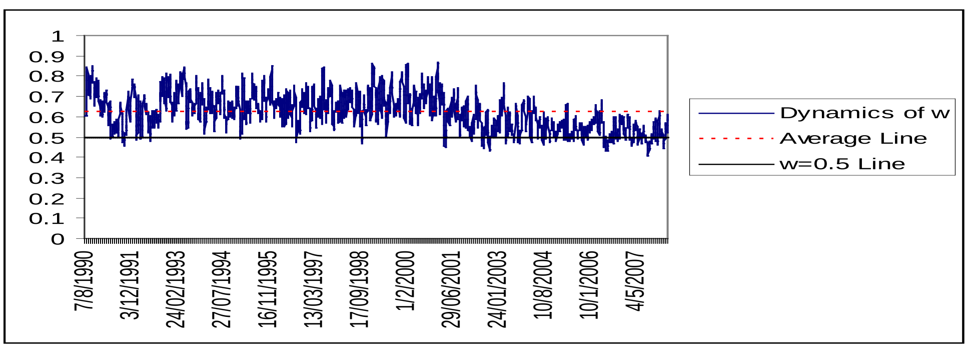
US-CA



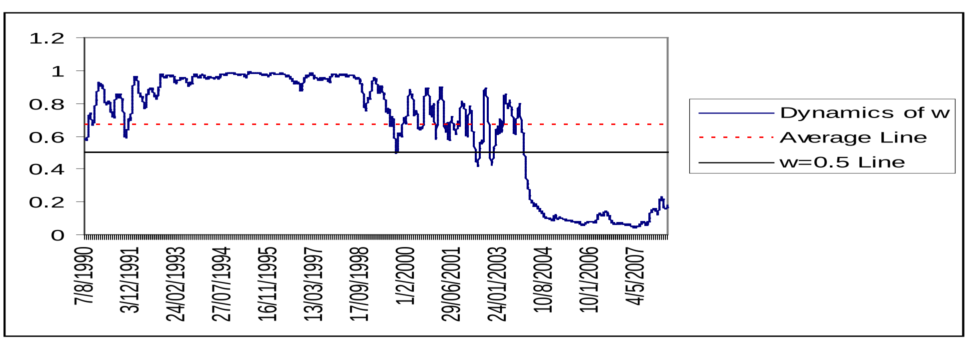
UK-GM



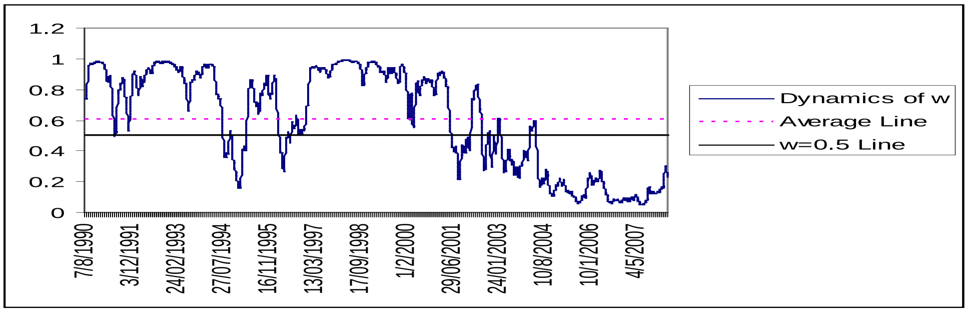
UK-FR



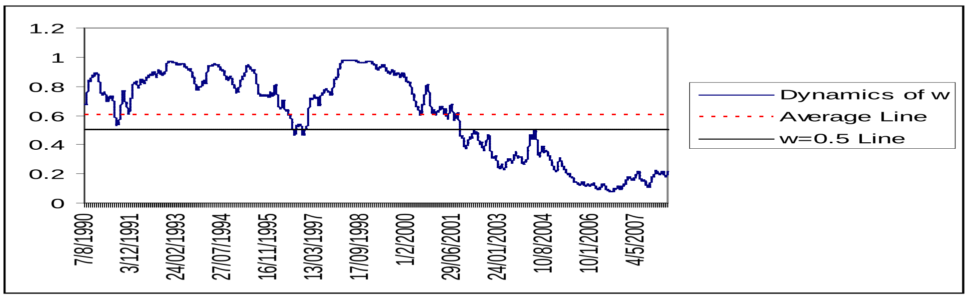
GM-FR



JP-KO



HK-KO



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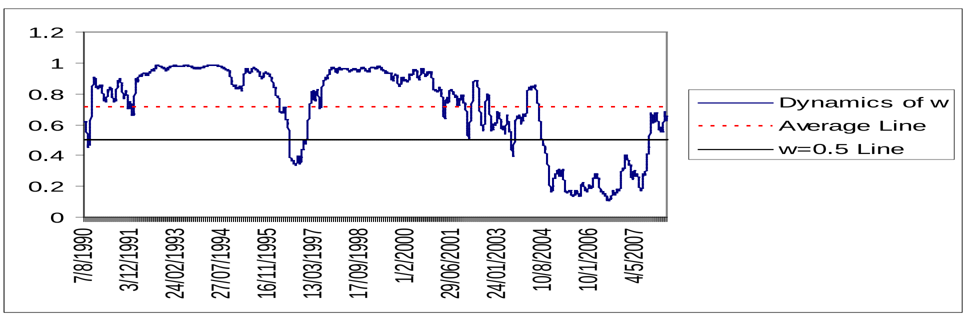
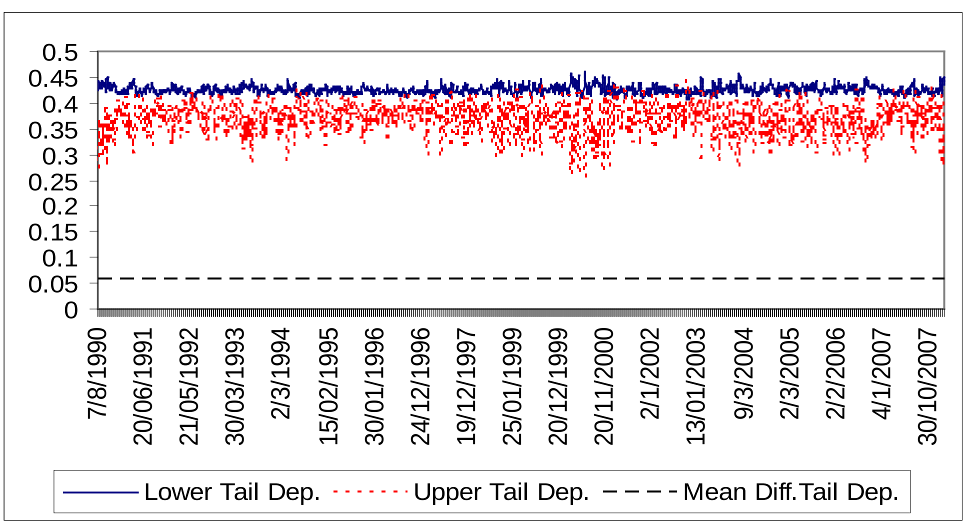
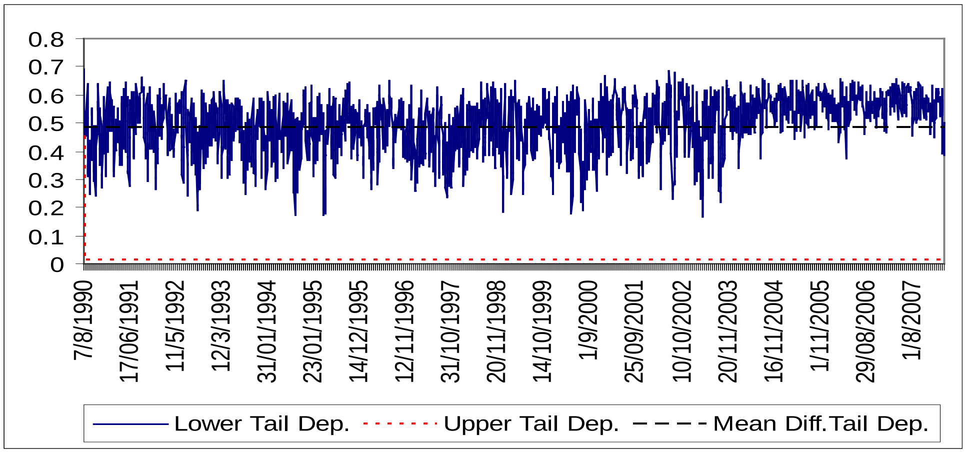


Figure 2: Dynamics of the Tail Dependence

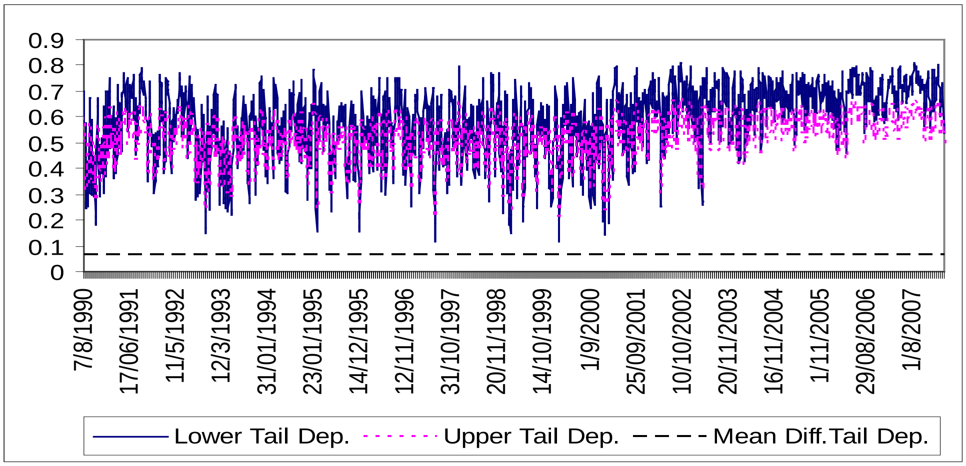
US-CA



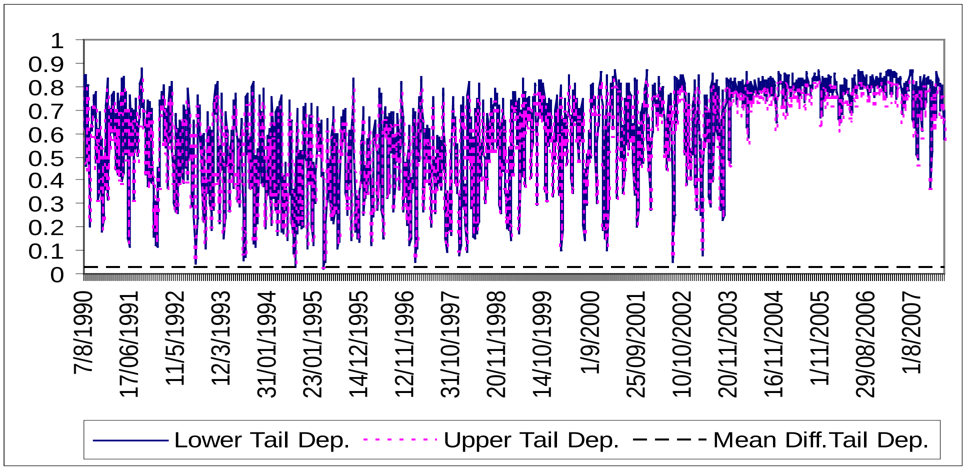
UK-GM



UK-FR



GM-FR



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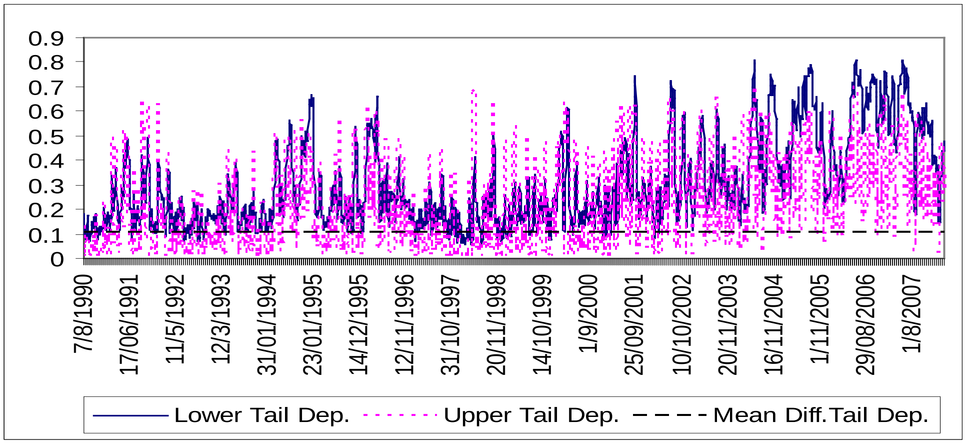

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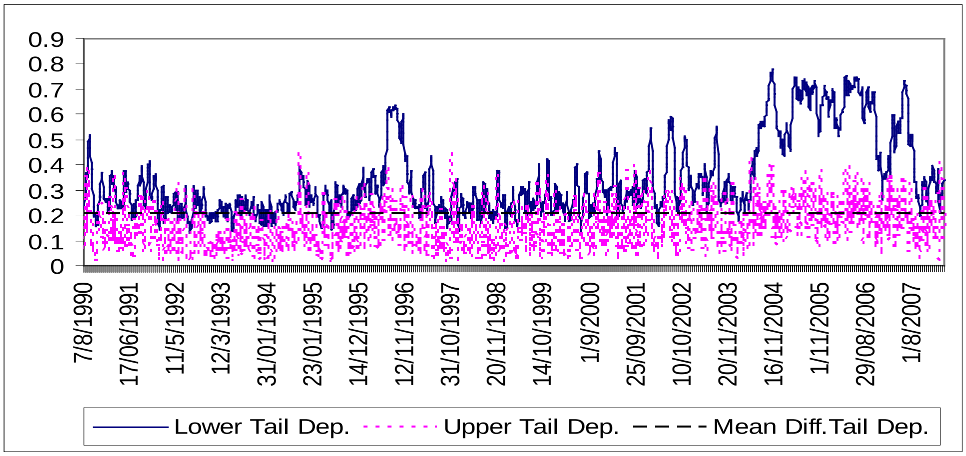

KO-HK